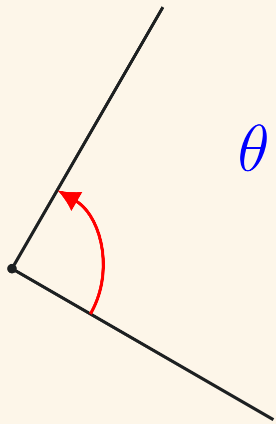
The Comprehensive List of

References in Geometry

AN ILLUSTRATED MANUAL



DECEMBER 2020

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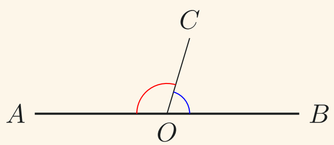
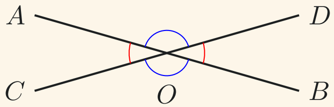
Appendix B

Lines



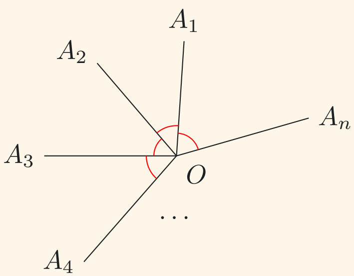
Adj. ∠s on st. line

$\angle AOC + \angle COB = 180^\circ.$



Vert. oppo. ∠s

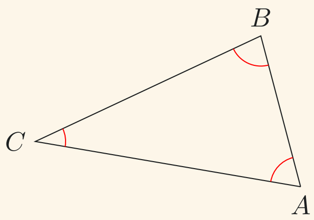
$\angle AOC = DOB,$
$\angle AOD = COB.$



∠s at a pt.

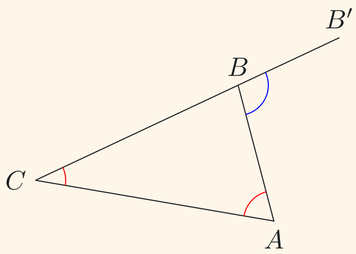
$$\angle A_1OA_2 + \angle A_2OA_3 + \cdots + \angle A_{n-1}OA_n = 360^\circ.$$

Triangles



$\angle$ sum of $\triangle$s

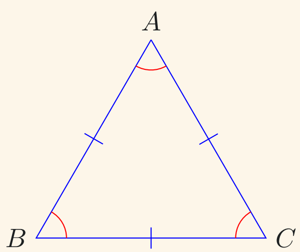
$$\angle CAB + \angle ABC + \angle BCA = 180^\circ.$$



Ext. $\angle$ of $\triangle$s

$$\angle CAB + \angle BCA = \angle B'BA.$$

Equilateral Triangles

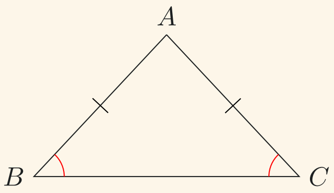


Prop. of equil. $\triangle$s

$$AB = BC = CA,$$

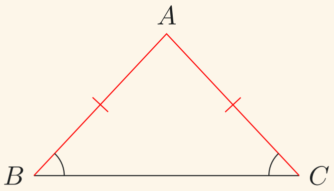
$$\angle CAB = \angle ABC = \angle BCA = 60^\circ.$$

Isosceles Triangles



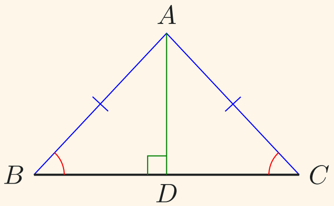
Base ∠s, isos. △

$$AB = AC$$
$$\Downarrow$$
$$\angle ABC = \angle ACB.$$



Sides oppo. eq. ∠s

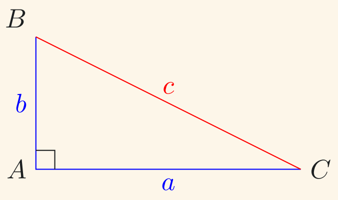
$$\angle ABC = \angle ACB$$
$$\Downarrow$$
$$AB = AC.$$



Prop. of isos. △s

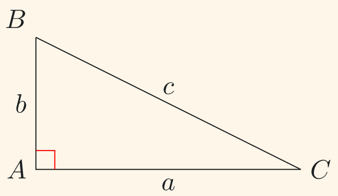
$$\angle ABC = \angle ACB$$
$$\Updownarrow$$
$$AB = AC$$
$$\Updownarrow$$
$$AD \perp BC.$$

Pythagoras's Theorem



Pyth. thrm.

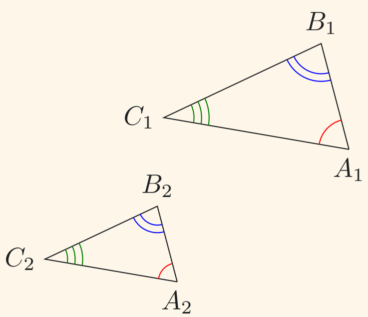
$$BA \perp AC$$
$$\Downarrow$$
$$a^2 + b^2 = c^2.$$



Conv. of Pyth. thrm.

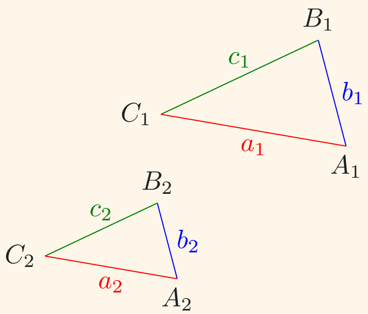
$$a^2 + b^2 = c^2$$
$$\Downarrow$$
$$BA \perp AC.$$

Similar Triangles



Corr. $\angle$_s_ | $\sim \triangle$_s_

$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$
$$\Downarrow$$
$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



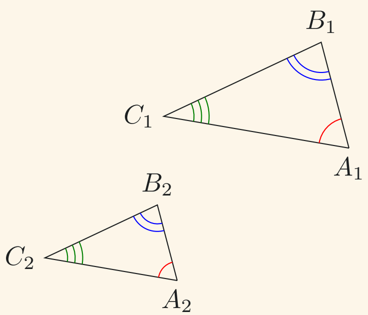
Corr. sides | ∼ △s

$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$

$$\Downarrow$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$
$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$
$$\frac{a_1}{b_1} = \frac{a_2}{b_2}.$$

Conditions for Proving Similar Triangles



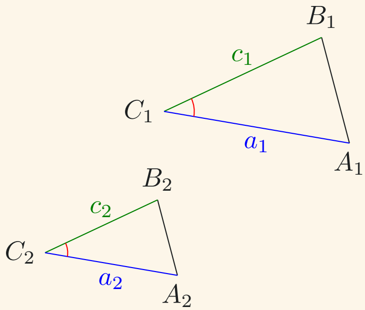
AAA

$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$

$$\Downarrow$$

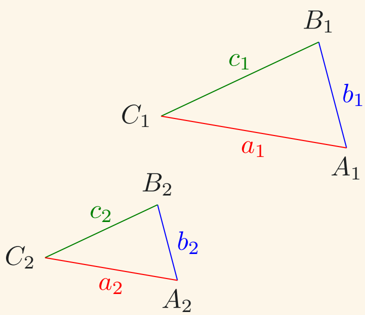
$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.$$

◀ Any two of the three is sufficient.



Ratio of two sides, incl. ∠

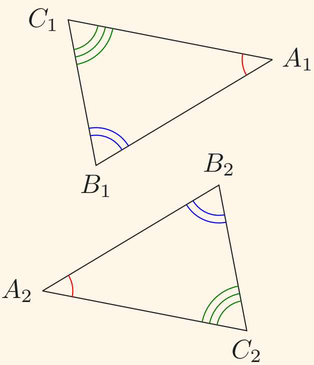
$$\frac{a_1}{c_1} = \frac{a_2}{c_2},$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$
$$\Downarrow$$
$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.$$



Three sides proportional

$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$
$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$
$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$
$$\Downarrow$$
$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.$$

Congruent Triangles

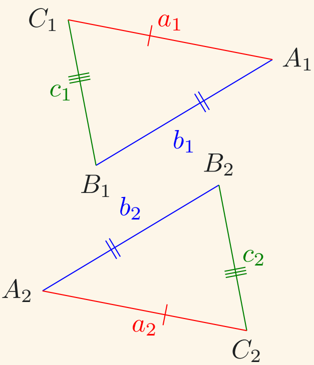


Corr. ∠s | ≅ △s

$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$

$$\Downarrow$$

$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



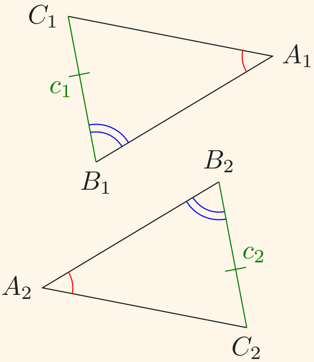
Corr. sides | ≅ △s

$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$

$$\Downarrow$$

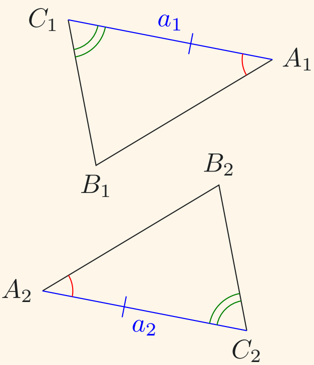
$$a_1 = a_2,$$
$$b_1 = a_2,$$
$$c_1 = a_2.$$

Conditions for Proving Congruent Triangles



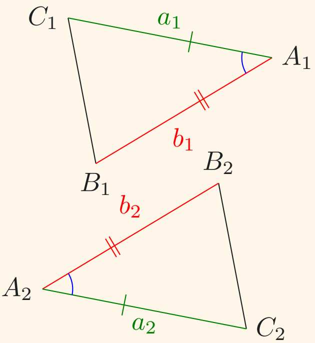
AAS

$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$c_1 = c_2$$
$$\Downarrow$$
$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.$$



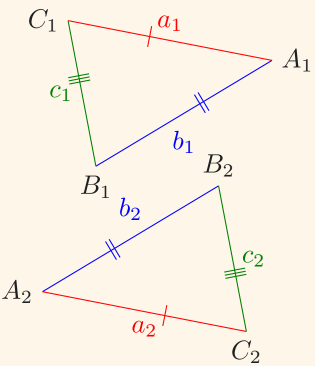
ASA

$$\angle A_1B_1C_1 = \angle C_2A_2B_2,$$
$$a_1 = a_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2,$$
$$\Downarrow$$
$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.$$



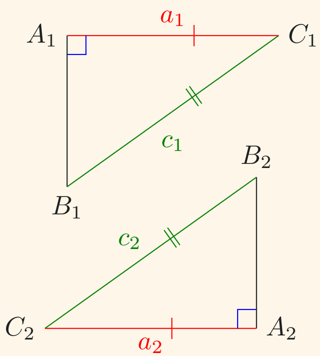
SAS

$$a_1 = b_1,$$
$$\angle B_1A_1C_1 = \angle B_2A_2C_2,$$
$$b_1 = b_2$$
$$\Downarrow$$
$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.$$



SSS

$$a_1 = a_2,$$
$$b_1 = a_2,$$
$$c_1 = a_2$$
$$\Downarrow$$
$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.$$

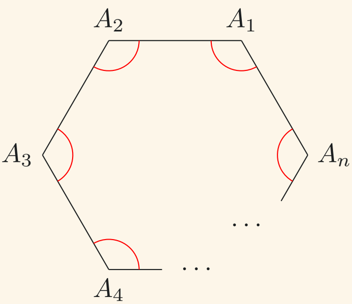


RHS

$$\angle B_1A_1C_1 = \angle B_2A_2C_2$$
$$= 90^\circ,$$
$$c_1 = c_2,$$
$$a_1 = a_2$$
$$\Downarrow$$
$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.$$

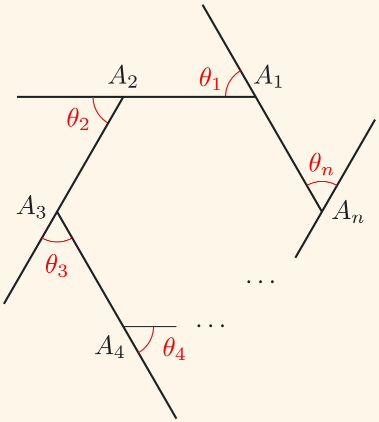
◀ One of the sides must be the hypotenuse.

Polygons



∠ sum of polygons

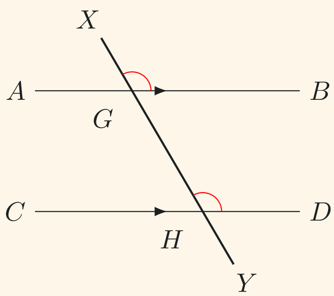
$$\angle A_nA_1A_2 + \angle A_1A_2A_3 + \cdots$$
$$+\angle A_{n-1}A_nA_1 = (n-2)\cdot 180^\circ.$$



Ext. ∠s of polygons

$$\theta_1 + \theta_2 + \cdots + \theta_n = 360^\circ.$$

Parallel Lines

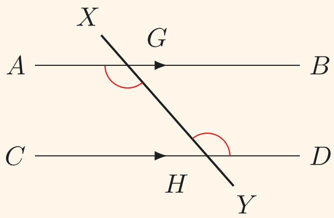


Corr.* ∠*s | $AB \,/\!/\, CD$

$$AB \,/\!/\, CD$$

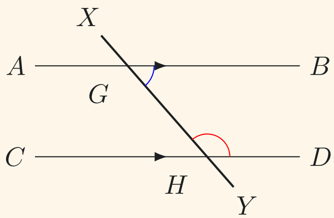
$$\Downarrow$$

$$\angle XGB = \angle XHD.$$



Alt. ∠s | $AB \parallel CD$

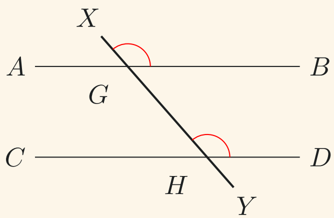
$$AB \parallel CD$$
$$\Downarrow$$
$$\angle YGA = \angle XHD.$$



Int. ∠s | $AB \parallel CD$

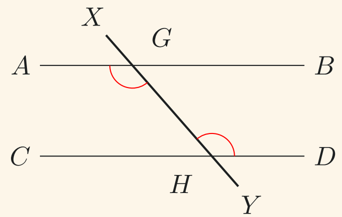
$$AB \parallel CD$$
$$\Downarrow$$
$$\angle YGB + \angle XHD = 180^\circ.$$

Conditions for Proving Parallelism



Int. ∠s eq.

$$\angle YGB + \angle XHD = 180^\circ$$
$$\Downarrow$$
$$\underline{\underline{AB \parallel CD}}.$$

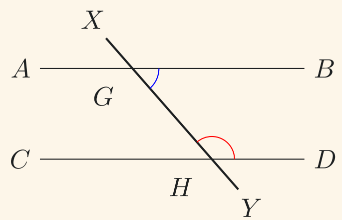


Alt. ∠s eq.

$\angle YGA = \angle XHD$

$\Downarrow$

$AB \parallel CD$.



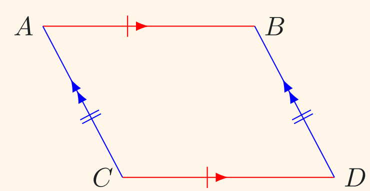
Int. ∠s eq

$\angle YGB + \angle XHD = 180^\circ$

$\Downarrow$

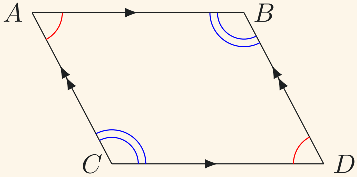
$AB \parallel CD$.

Parallelograms



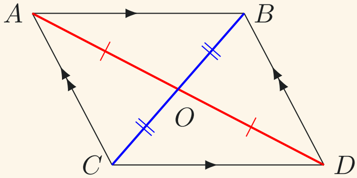
Oppo. sides of ∥-gram

$AB = CD$,
$AC = BD$.



Oppo. ∠s of //-gram

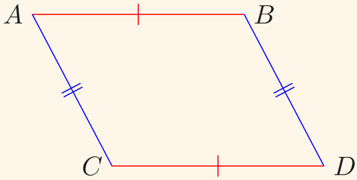
$\angle BAC = \angle BDC$,
$\angle ACD = \angle ABD$.



Diags. of //-gram

$OA = OD$,
$OB = OC$.

Conditions for Identifying Parallelograms

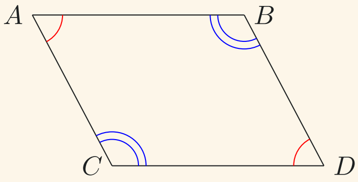


Oppo. sides eq.

$AB = CD$,
$AC = BD$

$\Downarrow$

$ABCD$ is a //-gram.

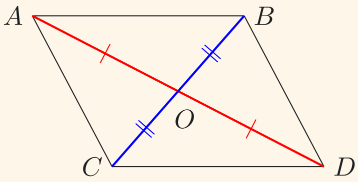


Oppo. ∠s eq.

$\angle BAC = \angle BDC$,
$\angle ACD = \angle ABD$

$$\Downarrow$$

$ABCD$ is a //-gram.

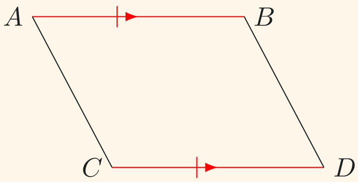


Diags. bisect each other

$OA = OD$,
$OB = OC$

$$\Downarrow$$

$ABCD$ is a //-gram.



Two sides eq. and //

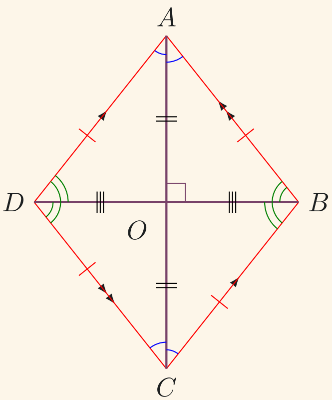
$AB = CD$,
$AB \parallel BD$

$$\Downarrow$$

$ABCD$ is a //-gram.

Other Quadrilaterals

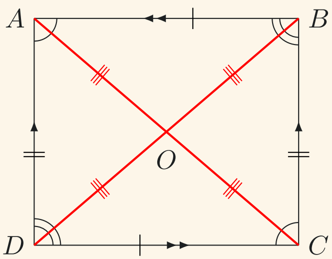
Rhombus - a quadrilateral with four equal sides



Prop. of rhombuses

$$AB \parallel CD \qquad AD \parallel BC,$$
$$OA = OC \qquad OB = OD,$$
$$AB = BC = CD = DA,$$
$$AC \perp BD,$$
$$\angle OAD = \angle OAB$$
$$= \angle OCD = \angle OCB,$$
$$\angle ODA = \angle ODC$$
$$= \angle OBA = \angle OBC.$$

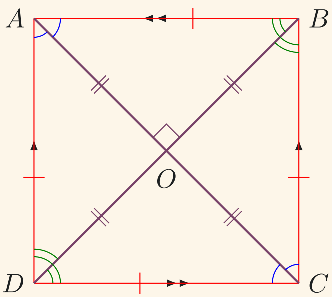
Rectangle - a quadrilateral with four equal interior angles



Prop. of rectangles

$$AB \parallel CD \qquad AD \parallel BC,$$
$$AB = CD \qquad AD = BC,$$
$$\angle BAD = \angle BCD,$$
$$\angle ABC = \angle ADC,$$
$$OA = OB = OC = OD.$$

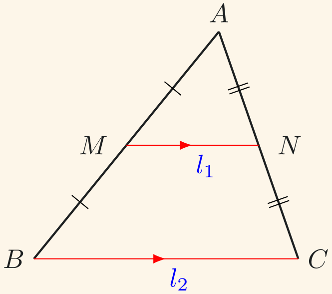
Square - a quadrilateral with four equal sides and interior angles



Prop. of squares

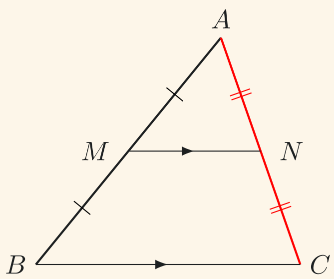
$$AB \parallel CD \qquad AD \parallel BC,$$
$$AB = BC = CD = DA,$$
$$OA = OB = OC = OD,$$
$$AC \perp BD,$$
$$\angle OAD = \angle OAB = 90^\circ$$
$$= \angle OCD = \angle OCB,$$
$$\angle ODA = \angle ODC = 90^\circ$$
$$= \angle OBA = \angle OBC.$$

Miscellaneous Results



Mid-pt. thrm.

$$MA = MA \quad NA = NC$$
$$\Downarrow$$
$$2l_1 = l_2,$$
$$MN \parallel BC.$$

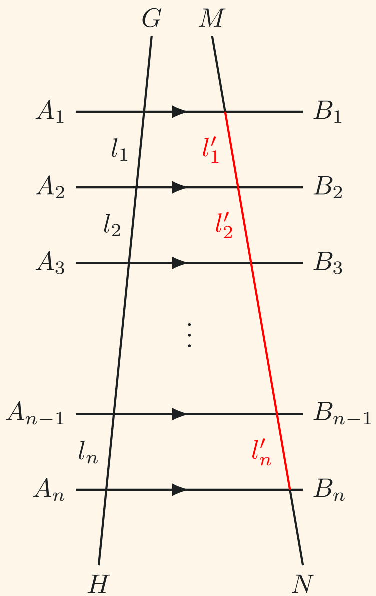


Intercept thrm. variant i

$$MA = MA \quad MN \parallel BC$$

$$\Downarrow$$

$$NA = NC.$$



Intercept thrm. variant ii

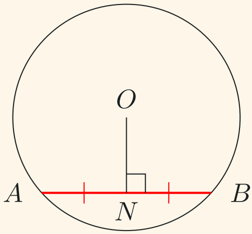
$$A_1B_1 \parallel A_2B_2 \parallel \cdots \parallel A_nB_n$$

$$l_1 = l_2 = \cdots = l_n$$

$$\Downarrow$$

$$l'_1 = l'_2 = \cdots = l'_n.$$

Circles

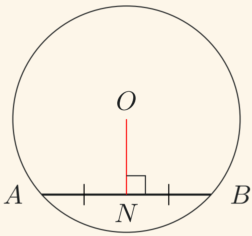


Line from cen. $\perp$ ***chord bisects chord***

$$ON \perp AB$$

$$\Downarrow$$

$$NA = NB.$$

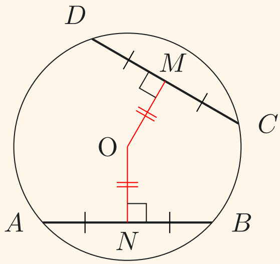


Line joining cen. to mid-pt. of chord $\perp$ ***chord***

$$NA = NB$$

$$\Downarrow$$

$$ON \perp AB.$$



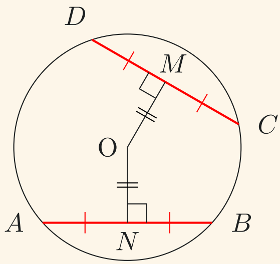
Eq. chords equidis. from cen.

$$ON \perp AB \quad OM \perp CD,$$

$$AB = CD$$

$$\Downarrow$$

$$ON = OM.$$



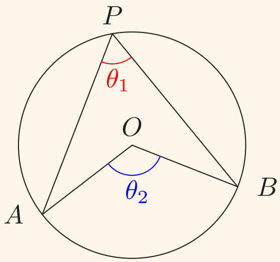
Chords equidis. from cen. are eq.

$$ON \perp AB \quad OM \perp CD,$$

$$ON = OM$$

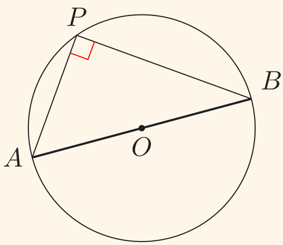
$$\Downarrow$$

$$AB = CD.$$



∠ *at cen. twice*
∠ *at* $\odot^{ce}$

$$2\theta_1 = \theta_2.$$

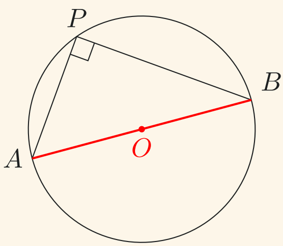


∠ *in semi-circ.*

AB is a diameter

$$\Downarrow$$

$$\angle APB = 90^\circ.$$

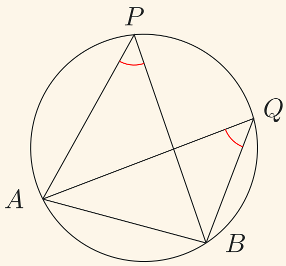


Converse of
∠ *in semi-circ.*

$$\angle APB = 90^\circ$$

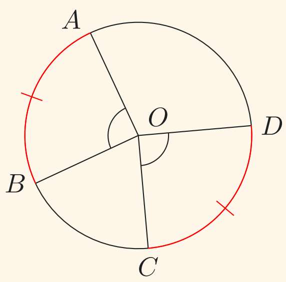
$$\Downarrow$$

AB is a diameter.



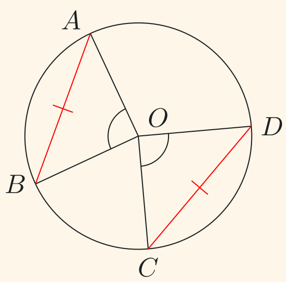
$\angle$s in the same segment

$\angle APB = \angle AQB.$



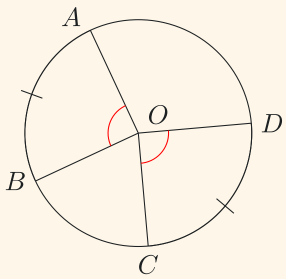
Eq. $\angle$s, eq. arcs

$$\angle AOB = \angle COD$$
$$\Downarrow$$
$$\overset{\frown}{AB} = \overset{\frown}{CD}.$$



Eq. $\angle$s, eq. chords

$$\angle AOB = \angle COD$$
$$\Downarrow$$
$$AB = CD.$$

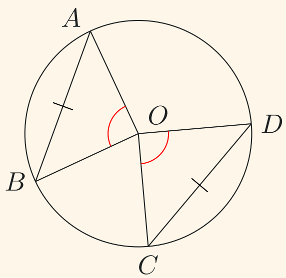


Eq. arcs, eq. $\angle$s

$$\overset{\frown}{AB} = \overset{\frown}{CD}$$

$$\Downarrow$$

$$\angle AOB = \angle COD.$$

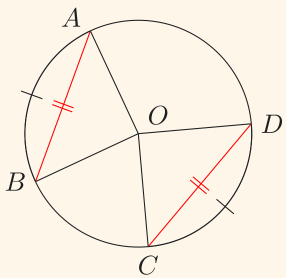


Eq. chords, eq. $\angle$s

$$AB = CD$$

$$\Downarrow$$

$$\angle AOB = \angle COD.$$

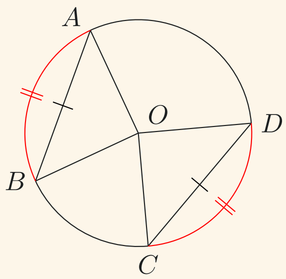


Eq. arcs, eq. chords

$$\overset{\frown}{AB} = \overset{\frown}{CD}$$

$$\Downarrow$$

$$AB = CD.$$

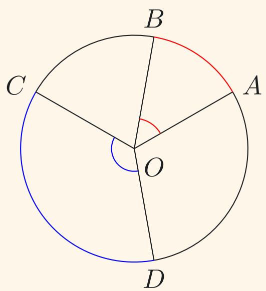


Eq. chords, eq. arcs

$$AB = CD$$

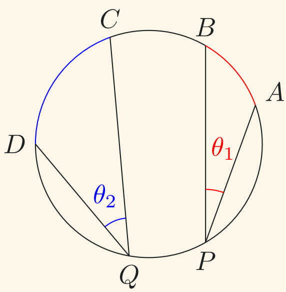
$$\Downarrow$$

$$\overset{\frown}{AB} = \overset{\frown}{CD}.$$



Arcs prop. to $\angle$s at cen.

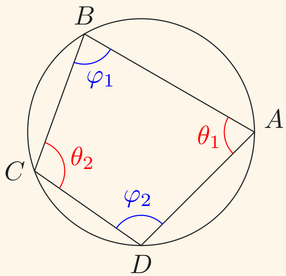
$$\frac{\overset{\frown}{AB}}{\overset{\frown}{CD}} = \frac{\theta_1}{\theta_2}.$$



Arcs prop. to $\angle$s at $\odot^{ce}$

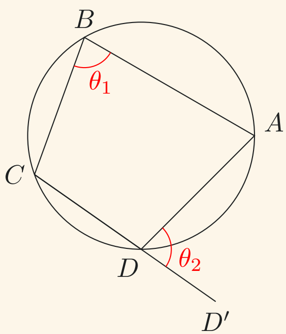
$$\frac{\overset{\frown}{AB}}{\overset{\frown}{CD}} = \frac{\theta_1}{\theta_2}.$$

Cyclic Quadrilaterals



Oppo. ∠s, cyclic quad.

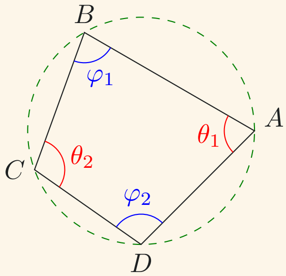
$$\theta_1 + \theta_2 = 180°,$$
$$\varphi_1 + \varphi_2 = 180°.$$



Ext. ∠s, cyclic quad.

$$\theta_1 = \theta_2.$$

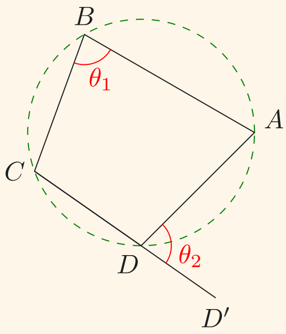
Conditions for Identifying Cyclic Quadrilaterals



Oppo. ∠s supp.

$$\theta_1 + \theta_2 = 180^\circ,$$
$$\varphi_1 + \varphi_2 = 180^\circ$$
$$\Downarrow$$

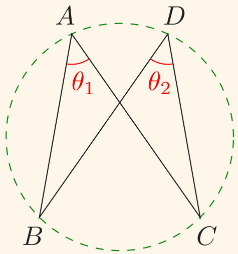
A, B, C and D are concylic.



Ext. ∠ = int. oppo. ∠

$$\theta_1 = \theta_2$$
$$\Downarrow$$

A, B, C and D are concylic.



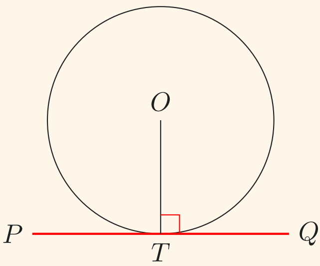
Converse of ∠s in the same segment

$\theta_1 = \theta_2$

$\Downarrow$

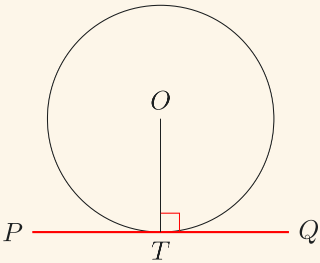
A, B, C and D are concylic.

Tangents



Tan. ⊥ radius

$OT \perp PQ$.

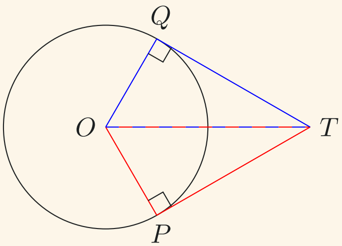


Converse of tan. ⊥ radius

$OT \perp PQ$.

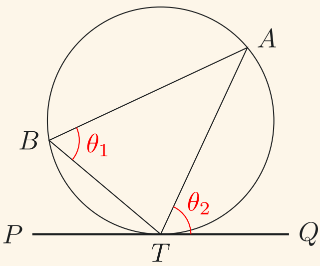
$\Downarrow$

PQ is tangent to the circle at T.



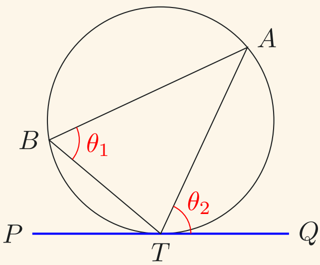
Tan. prop.

$\triangle OPT \cong \triangle OQT$.



∠ in alt. segment

$\theta_1 = \theta_2$.



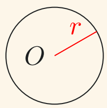
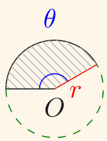
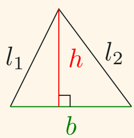
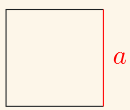
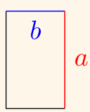
Converse of ∠ in alt. segment

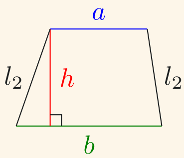
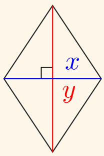
$\theta_1 = \theta_2$

$\Downarrow$

PQ is tangent to the circle at T.

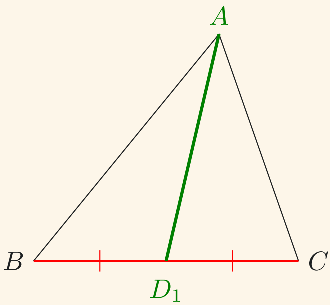
Appendix A: Area and Perimeters of Common Plane Figures

FIGURE	AREA	PERIMETER	
	πr^2	$2\pi r$	◀ CIRCLES
	$\frac{\pi\theta r^2}{360°}$	$\frac{2\pi\theta r}{360°} + 2r$	◀ SECTORS
	$\frac{hb}{2}$	$b + l_1 + l_2$	◀ TRIANGLES
	a^2	$4a$	◀ SQUARES
	ab	$2(a+b)$	◀ RECTANGLES

	$\frac{h\,(a+b)}{2}$	$a+b+l_1+l_2$	◀ Trapeziums
	$\frac{xy}{2}$	$2\sqrt{x^2+y^2}$	◀ Rhombuses

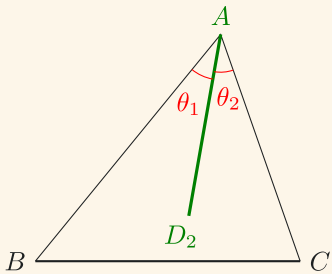
Appendix B: Special Points and Lines in △s

AD_1 is the *median* of BC



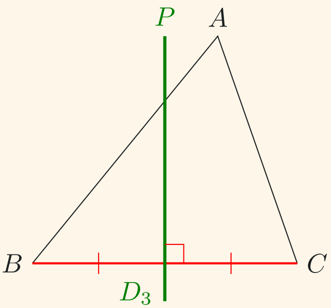
$BD_1 = CD_1$.

AD_2 is an *angle bisector* of $\angle BAC$



$\theta_1 = \theta_2$.

PD_3 is a *perpendicular bisector* of BC

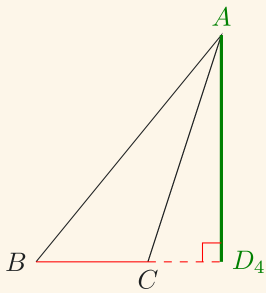


$BD_3 = CD_3$,

$PD_3 \perp BC$.

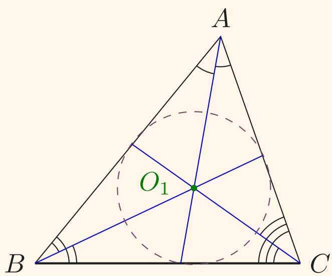
AD_2 is the *altitude* of BC

◀ May not necessarily lie in △.



$\theta_1 = \theta_2$.

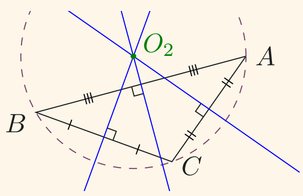
O_1 is the *in-centre* of $\triangle ABC$



The *in-centre* is the point of intersection of the three angle bisectors (of a triangle). It is the center of the triangle's inscribed circle.

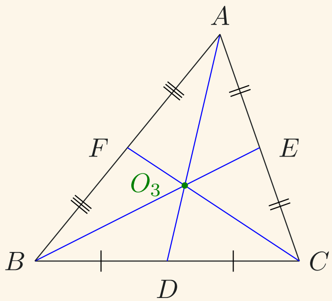
O_2 is the *circumcentre* of $\angle BAC$

◀ May not necessarily lie in △.



The *circumcentre* is the point of intersection of the three perpendicular bisectors. It is the center of the triangle's circumcircle.

O_1 is the *centroid* of $\triangle ABC$

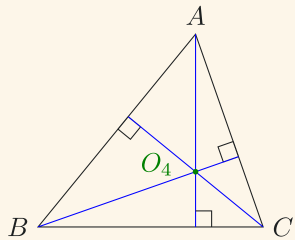


The *centroid* is the point of intersection of the three medians (of a triangle).
It divides each median in the ratio of 2 : 1, i.e.,

$$\frac{O_3A}{O_3D} = \frac{O_3B}{O_3E} = \frac{O_3C}{O_3F} = 2.$$

O_4 is the *orthocentre* of $\triangle ABC$

◀ May not necessarily lie in △.



The *orthocentre* is the point of intersection of the three altitudes.

These four points are generally distinct from one another.